Holographic Hall System with Momentum Relaxation (DC Transport Coefficient from Black Hole Horizon)

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Based on on-going work
in collaboration with Sang-Jin Sin, Keun-Young Kim and Kyung Kiu Kim

- Transport coefficient
  - Response of the electrical current $\vec{J}$ and the heat current $\vec{Q}$ to an applied electric field $\vec{E}$ and a temperature gradient $\vec{\nabla}T$
  - Transport coefficient

$$\begin{pmatrix} \langle J_i \rangle \\ \langle Q_i \rangle \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij}T \\ \bar{\alpha}_{ij}T & \bar{\kappa}_{ij}T \end{pmatrix} \begin{pmatrix} E_j \\ -(\nabla_j T)/T \end{pmatrix}$$

σ_{ij}: Electric conductivity (σ_{xy}: Hall conductivity)

α_{ij}, $\bar{\alpha}_{ij}$: Thermoelectric conductivity

$\bar{\kappa}_{ij}$: Thermal conductivity

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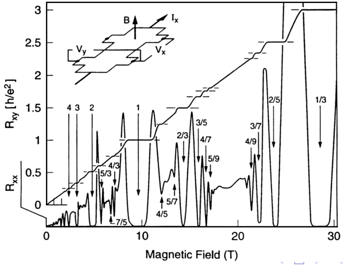
  - Fractional quantum Hall effect



- Transport coefficient
  - Nernst response: Electric field induced by a thermal gradient, $E_i = -\vartheta_{ij}\nabla_j T$

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$e_N \equiv \vartheta_{yx}$: Nernst signal

$\nu = e_N/B$: Nernst coefficient

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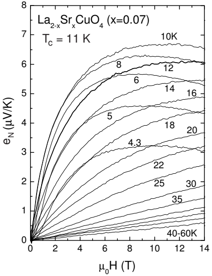

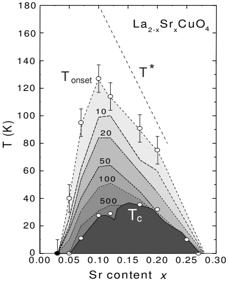

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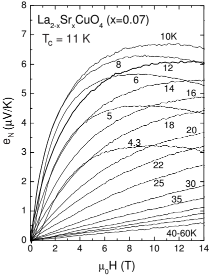

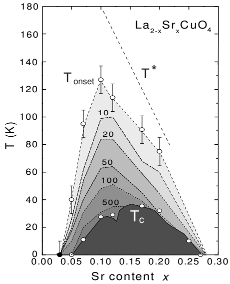

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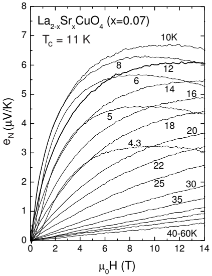

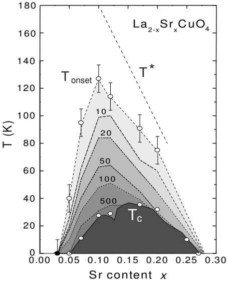

- Performing explicit calculations of strongly interacting transport is extremely hard
- We apply holography method to understand such interesting phenomena

- General action with metric, $U(1)$ gauge field, dilaton and "axion"

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi_1(\phi)(\partial\chi_1)^2 + \Phi_2(\phi)(\partial\chi_2)^2\right] - V(\phi) - \frac{Z(\phi)}{4}F^2\right],$$

$$(\Phi_i, Z(\phi) \geq 0)$$

- Einstein and field equations

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{L} - \frac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}\sum_{i=1,2}\Phi_i(\phi)(\partial_\mu\chi_i)(\partial_\nu\chi_i) - \frac{Z(\phi)}{2}F_\mu^{\lambda}F_{\lambda\nu} &= 0 \\ \nabla_\mu\left(Z(\phi)F^{\mu\nu}\right) &= 0 \\ \nabla_\mu\left(\Phi_i(\phi)\nabla^\mu\chi_i\right) &= 0 \\ \nabla^2\phi - \frac{1}{2}\sum_{i=1,2}\frac{\partial\Phi_i}{\partial\phi}(\partial\chi_i)^2 - \frac{\partial V(\phi)}{\partial\phi} - \frac{1}{4}F^2\frac{\partial Z(\phi)}{\partial\phi} &= 0 \end{aligned}$$

- Ansatz for background

$$ds^2 = -U(r)dt^2 + \frac{1}{U(r)}dr^2 + e^{v_1}dx^2 + e^{v_2}dy^2,$$

$$\chi_1 = k_1 x, \qquad \chi_2 = k_2 y$$

$$A = a(r)dt + \frac{B}{2}(xdy - ydx)$$

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- Horizon at $r = r_0$

$$U \sim T(r - r_0) + \cdots, \qquad a(r) \sim a_0(r - r_0) + \cdots$$

$$v_i \sim v_{i0} + \cdots, \qquad \phi \sim \phi_0 + \cdots \tag{1}$$

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- Symmetric gauge of the gauge potential

$$\Phi_1(\phi) = \Phi_2(\phi) = \Phi(\phi), \qquad v_1(r) = v_2(r) = v(r), \qquad k_1 = k_2 = k$$

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Example:

$\phi(r) = \text{constant}, \ \Phi(\phi) = 1, \ Z(\phi) = 1, \ V(\phi) = 6$

$\rightarrow$ Dyonic black hole with momentum relaxation

- Fluctuations (2014, Donos and Gauntlett)

$$
\begin{aligned}
\delta A_{x_i} &= \{-E_{x_i} + \zeta_{x_i} a(r)\}\, t + \delta a_{x_i}(r) \\
\delta G_{tx_i} &= -\zeta_{x_i} U(r)\, t + \delta g_{tx_i}(r) \\
\delta G_{rx_i} &= e^{v(r)} \delta g_{rx_i}(r) \\
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  - Time dependent term contains external source
    E_{x_i} : External electric field
    $\zeta_{x_i} = -\nabla_{x_i} T/T$: External thermal gradiant
  - all time dependence in the linearized equations disappears by imposing background equations of motion

- Electric current

$$
\begin{aligned}
J_{x_i} &\equiv Z(\phi)\sqrt{-g}F^{x_i r} \\
&= -Z(\phi)\left\{U(r)(-\epsilon_{ij}B\delta g_{rx_j}(r) + \delta a'_{x_i}(r)) + a'(r)\delta g_{tx_i}\right\}
\end{aligned}
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At the boundary ($r \to \infty$), J_{x_i} becomes electric current along x_i direction

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At the boundary ($r \to \infty$), J_{x_i} becomes electric current along x_i direction

- Maxwell equation

$$0 = \partial_\mu\left(Z(\phi)\sqrt{-g}F^{x_i\mu}\right)$$
$$= \partial_r\left(Z(\phi)\sqrt{-g}F^{x_i r}\right) + \partial_t\left(Z(\phi)\sqrt{-g}F^{x_i t}\right)$$
$$= \partial_r J_{x_i} - B\epsilon_{ij}e^{-v(r)}\zeta_{x_j}Z(\phi),$$

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\end{aligned}
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- Boundary current

$$
\begin{aligned}
J_{x_i}(\infty) &= J_{x_i}(r_0) + B\epsilon_{ij}\zeta_{x_j}\int_{r_0}^{\infty} dr' e^{-v(r')}Z(\phi(r')) \\
&\equiv J_{x_i}(r_0) + B\epsilon_{ij}\zeta_{x_j}\Sigma_1
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- Heat current

$$Q_{x_i} \equiv U^2(r)\,\partial_r\left(\frac{\delta g_{tx_i}(r)}{U(r)}\right) - a(r)J_{x_i}$$

At the boundary($r \to \infty$), $Q_{x_i} \to T_{tx_i} - \mu J_{x_i}$: heat current

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$$\begin{aligned} Q_{x_i}(\infty) &= Q_{x_i}(r_0) + B\epsilon_{ij}E_{x_j}\int_{r_0}^{\infty} dr' e^{-v(r')}Z(\phi(r')) \\ &- 2B\epsilon_{ij}\zeta_{x_j}\int_{r_0}^{\infty} dr' a(r')e^{-v(r')}Z(\phi(r')) \\ &\equiv Q_{x_i}(r_0) + B\epsilon_{ij}E_{x_j}\Sigma_1 - B\epsilon_{ij}\zeta_{x_j}\Sigma_2. \end{aligned} \tag{2}$$

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- Boundary currents J_{x_i}, Q_{x_i} can be written in terms of functions at the horizon and definite integration from black hole horizon to boundary

- Regularity at the black hole horizon

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$$\delta a_{x_i}(r) \sim -\frac{E_{x_i}}{4\pi T}\ln(r-r_0)+\cdots$$

$$\delta g_{tx_i}(r) \sim \delta g^{(0)}_{tx_i} + \delta g^{(1)}_{tx_i}(r-r_0)+\cdots$$

$$\delta g_{rx_i}(r) \sim e^{-v(r_0)}\frac{\delta g^{(0)}_{tx_i}}{U(r_0)}+\cdots$$

$$\delta \chi_{x_i}(r) \sim \chi^{(0)}_{x_i} + \chi^{(1)}_{x_i}(r-r_0)+\cdots$$

DC Transport from Horizon Data

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- Boundary currents

$$J_{x_i}(\infty) = -(Qe^{-v_0}+B\epsilon_{ij}e^{-v_0}Z_0)\delta g^{(0)}_{tx_i} + E_{x_i}Z_0 + B\epsilon_{ij}\zeta_{x_j}\Sigma_1$$

$$Q_{x_i}(\infty) = -4\pi T\delta g^{(0)}_{tx_i} + B\epsilon_{ij}E_{x_j}\Sigma_1 - B\epsilon_{ij}\zeta_{x_j}\Sigma_2$$

DC Transport from Horizon Data

- Transport coefficients

$$\sigma^{ij} = \frac{\partial J_{x_i}(\infty)}{\partial E_{x_j}} = -(Qe^{-v_0} + B\epsilon_{ij}e^{-v_0}Z_0)\frac{\partial \delta g^{(0)}_{tx_i}}{\partial E_{x_j}} + Z_0\,\delta_{ij}$$

$$\alpha^{ij} = \frac{1}{T}\frac{\partial J_{x_i}(\infty)}{\partial \zeta_{x_j}} = -(Qe^{-v_0} + B\epsilon_{ij}e^{-v_0}Z_0)\frac{1}{T}\frac{\partial \delta g^{(0)}_{tx_i}}{\partial \zeta_{x_j}} + \epsilon_{ij}\frac{B}{T}\Sigma_1$$

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- Einstein equations at the horizon

$$\delta g^{(0)}_{tx} = \frac{e^{v_0}}{B^2 Z_0 + k^2 e^{v_0}\Phi_0}\left[BQe^{-v_0}\delta g^{(0)}_{ty} - BZ_0E_y - QE_x - 4\pi e^{v_0}T\zeta_x\right]$$

$$\delta g^{(0)}_{ty} = \frac{e^{v_0}}{B^2 Z_0 + k^2 e^{v_0}\Phi_0}\left[-BQe^{-v_0}\delta g^{(0)}_{tx} - BZ_0E_x - QE_y - 4\pi e^{v_0}T\zeta_y\right]$$

- Electric transport coefficients

$$\sigma^{xx} = \sigma^{xy} = \frac{e^{v_0} k^2 \Phi_0 \left(Q^2 + B^2 Z_0^2 + e^{v_0} k^2 Z_0 \Phi_0\right)}{B^2 Q^2 + \left(B^2 Z_0^2 + e^{v_0} k^2 \Phi_0\right)^2}$$

$$\sigma^{xy} = -\sigma^{yx} = \frac{BQ \left(Q^2 + B^2 Z_0^2 + 2e^{v_0} k^2 Z_0 \Phi_0\right)}{B^2 Q^2 + \left(B^2 Z_0^2 + e^{v_0} k^2 \Phi_0\right)^2}$$

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- Hall angle

$$\tan\theta_H = \frac{\sigma^{xy}}{\sigma^{xx}} = \frac{BQ \left(Q^2 + B^2 Z_0^2 + 2e^{v_0} k^2 Z_0 \Phi_0\right)}{e^{v_0} k^2 \Phi_0 \left(Q^2 + B^2 Z_0^2 + e^{v_0} k^2 Z_0 \Phi_0\right)}$$

- Thermoelectric transport coefficients

$$\alpha^{xx} = \alpha^{yy} = \frac{4\pi e^{2v_0} k^2 Q \Phi_0}{B^2 Q^2 + \left(B^2 Z_0^2 + e^{v_0} k^2 \Phi_0\right)^2}$$

$$\alpha^{xy} = -\alpha^{yx} = \frac{4\pi e^{v_0} B \left(Q^2 + B^2 Z_0^2 + e^{v_0} k^2 Z_0 \Phi_0\right)}{B^2 Q^2 + \left(B^2 Z_0^2 + e^{v_0} k^2 \Phi_0\right)^2} + \frac{B}{T} \Sigma_1,$$

  where

$$\Sigma_1 = \int_{r_0}^{\infty} dr' e^{-v(r')} Z(\phi(r'))$$

- $\alpha^{ij} = \bar{\alpha}^{ij}$

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- $\alpha^{ij} = \bar{\alpha}^{ij}$

- Nernst signal

$$e_N = -(\sigma^{-1} \cdot \alpha)^{yx}$$

$$= \frac{e^{v_0} k^2 B}{T} \cdot \frac{4\pi T e^{v_0} Z_0^2 \Phi_0 + \Phi_0 \Sigma_1 \left(Q^2 + B^2 Z_0^2 + e^{v_0} k^2 Z_0 \Phi_0\right)}{Q^4 + 2e^{v_0} k^2 Q^2 Z_0 \Phi_0 + Z_0^2 \left(B^2 Q^2 + e^{2v_0} k^4 \Phi_0^2\right)}$$

- Thermal transport coefficients

$$\bar{\kappa}^{xx} = \bar{\kappa}^{yy} = \frac{16\pi^2 T e^{2v_0} \left(B^2 Z_0 + e^{v_0} k^2 \Phi_0\right)}{B^2 Q^2 + \left(B^2 Z_0^2 + e^{v_0} k^2 \Phi_0\right)^2}$$

$$\bar{\kappa}^{xy} = -\bar{\kappa}^{yx} = \frac{16\pi^2 T e^{2v_0} BQ}{B^2 Q^2 + \left(B^2 Z_0^2 + e^{v_0} k^2 \Phi_0\right)^2} - \frac{B}{T}\Sigma_2$$

where

$$\Sigma_2 = 2\int_{r_0}^{\infty} dr' a(r') e^{-v(r')} Z(\phi(r')).$$

- Dyonic black hole solution with momentum relaxation

$\phi = \text{const.}$, $\Phi_1 = \Phi_2 = 1$, $V(\phi) = -\frac{6}{L^2}$, $Z(\phi) = 1$

$$S_0 = \frac{1}{16\pi G}\int_M d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - \frac{1}{2}\sum_{I=1}^{2}(\partial\chi_I)^2\right] - \frac{1}{8\pi G}\int_{\partial M} d^3x\sqrt{-\gamma}K$$

$$U(r) = r^2 - \frac{\beta^2}{2} - \frac{m_0}{r} + \frac{\mu^2 + q_m^2}{4}\frac{r_0^2}{r^2} \ , \ e^{v_1(r)} = e^{v_2(r)} = r^2 \ ,$$

$$k_1 = k_2 = \beta \quad , \quad a(r) = \mu\left(1 - \frac{r_0}{r}\right) \quad , \quad B = q_m r_0$$

$$m_0 = r_0^3\left(1 + \frac{\mu^2 + q_m^2}{4r_0^2} - \frac{\beta^2}{2r_0^2}\right)$$

$$T = T_H = \frac{U'(r_0)}{4\pi} = \frac{1}{4\pi}\left(3r_0 - \frac{\mu^2 + q_m^2 + 2\beta^2}{4r_0}\right)$$

- Electric conductivities

$$\sigma^{xx} = r_0^2\beta^2 \cdot \frac{B^2 + r_0^2\left(\mu^2 + \beta^2\right)}{r_0^2\mu^2 B^2 + \left(B^2 + r_0^2\beta^2\right)^2}$$

$$\sigma^{xy} = r_0\mu B \cdot \frac{B^2 + r_0^2\left(\mu^2 + 2\beta^2\right)}{r_0^2\mu^2 B^2 + \left(B^2 + r_0^2\beta^2\right)^2}$$

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Clean limit $(\beta \to 0)$

$$\sigma^{xx} = 0, \qquad \sigma^{xy} = \frac{r_0\mu}{B} = \frac{Q}{B}$$

Electric limit$(B \to 0)$

$$\sigma^{xx} = 1 + \frac{\mu^2}{\beta^2}, \qquad \sigma^{xy} = 0$$

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Hall angle

$$\tan\theta_H \equiv \frac{\sigma^{xy}}{\sigma^{xx}} = \frac{\mu B}{r_0\beta^2} \cdot \frac{B^2 + r_0^2(\mu^2+2\beta^2)}{B^2 + r_0^2(\mu^2+\beta^2)}$$

- Thermoelectric conductivities

$$\alpha^{xx} = \frac{4\pi r_0^5 \beta^2 \mu}{r_0^2 \mu^2 B^2 + \left(B^2 + r_0^2 \beta^2\right)^2}$$

$$\alpha^{xy} = 4\pi r_0^3 B \cdot \frac{B^2 + r_0^2 \left(\mu^2 + \beta^2\right)}{r_0^2 \mu^2 B^2 + \left(B^2 + r_0^2 \beta^2\right)^2} + \frac{B}{T}$$

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- Nernst signal

$$e_N = \frac{\beta^2 B}{r_0 T} \cdot \frac{B^2 + r_0^2(4\pi r_0 T + \mu^2 + \beta^2)}{\mu^2 B^2 + r_0^2(\mu^2 + \beta^2)^2}$$

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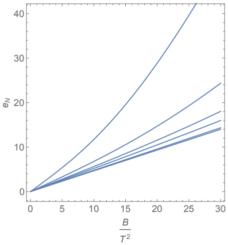
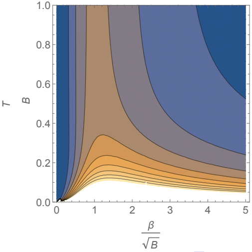




- Thermal conductivities

$$\bar{\kappa}^{xx} = 16\pi r_0^4 T \cdot \frac{B^2 + r_0^2\beta^2}{r_0^2\mu^2 B^2 + \left(B^2 + r_0^2\beta^2\right)^2}$$

$$\bar{\kappa}^{xy} = \frac{16\pi^2 r_0^5 \mu T B}{r_0^2\mu^2 B^2 + \left(B^2 + r_0^2\beta^2\right)^2} - \frac{\mu B}{T}.$$

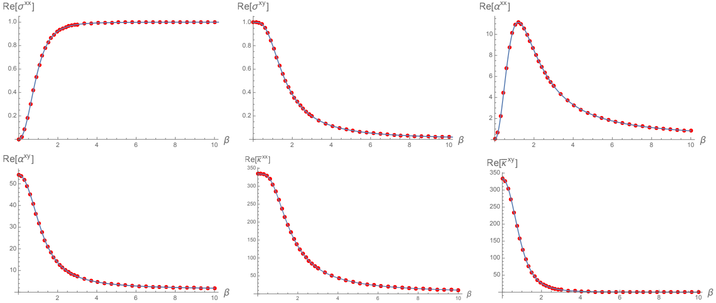
Dyonic Black Hole with Momentum Relaxation

- Thermal conductivities

$$\bar{\kappa}^{xx} = 16\pi r_0^4 T \cdot \frac{B^2 + r_0^2\beta^2}{r_0^2\mu^2 B^2 + \left(B^2 + r_0^2\beta^2\right)^2}$$

$$\bar{\kappa}^{xy} = \frac{16\pi^2 r_0^5 \mu T B}{r_0^2\mu^2 B^2 + \left(B^2 + r_0^2\beta^2\right)^2} - \frac{\mu B}{T}.$$

- Comparison to numerical calculation



Conlusion

- We derive general DC transport coefficients in general background with momentum relaxation by requiring regularity condition at the black hole horizon
- We apply our general formula to the dyonic black hole background and calculation DC transport coefficient
- We find dyonic black hole give large value of Nernst signal which indicates the existence of vortex fluid above T_C
- We check DC transport results exactly match numerical calculation from different approach
- We also calculate AC transport coefficient and investigating impurity effect of cyclotron frequency.

Thank you !!!